Formula Sheet PHYC/ECE 463 Advanced Optics 1 UNM (MTII)

Harmonic wave: $E = Re\{E_0 \exp(-i\omega t + ik \cdot r + \phi)\}$ $k = \frac{n\omega}{c} = \frac{2\pi n}{\lambda_0}$; $\frac{1}{\mu_0 \mu \varepsilon_0} = c^2$

Poynting vector $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$ $\vec{B} = \frac{1}{\omega}\vec{k} \times \vec{E} = \frac{1}{c}\hat{k} \times \vec{E}$ ($\hat{k}$ unit vector $\parallel \vec{k}$); Irradiance: $I = \langle S \rangle = \frac{\varepsilon_0 n c}{2}|E_0|^2$

Hermite-Gaussian Beams

$$\frac{E(x,y,z)}{E_0} = H_m\left(\frac{\sqrt{2}x}{w(z)}\right) H_p\left(\frac{\sqrt{2}y}{w(z)}\right) \frac{w_0}{w(z)} exp\left(-i\frac{kr^2}{2q(z)}\right) \times exp\left(-i\left[kz - (1+m+p)tan^{-1}\left(\frac{z}{z_0}\right)\right]\right), \begin{cases} H_0(u) = 1; H_1(u) = 2u \\ H_2(u) = 2(2u^2 - 1) \end{cases}$$

Divergence

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi n w^2(z)}; \quad w^2(z) = w_0^2\left(1 + \frac{z^2}{z_0^2}\right); \quad R(z) = z\left(1 + \frac{z_0^2}{z^2}\right); \quad z_0 = \frac{\pi n w_0^2}{\lambda_0}; \quad \theta_0 = \frac{2\lambda}{\pi\omega_0}$$

Fresnel Equations

$$r_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \qquad t_\parallel = \frac{2n_i \cos\theta_i}{n_t \cos\theta_i + n_i \cos\theta_t} = \frac{2\sin(\theta_t)\cos(\theta_i)}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

$$r_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \qquad t_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} = \frac{2\sin(\theta_t)\cos(\theta_i)}{\sin(\theta_i + \theta_t)}$$

Reflectance $R = |r|^2$; $1 = |r|^2 + \frac{n_t}{n_r}\frac{\cos\theta_t}{\cos\theta_r}|t|^2$; $R + T = 1$;

Snell: $n_i sin\theta_i = n_t sin\theta_t$
critical/pol. angles:
$sin\theta_C = tan\theta_B = n_2/n_1$

Complex index: $\tilde{n} = n + i\kappa \equiv \sqrt{\varepsilon/\varepsilon_0} = \sqrt{1-\chi}$ in the above expressions

Absorption coefficient (α) and skin depth (δ): $\alpha = \frac{2}{\delta} = \frac{4\pi\kappa}{\lambda_0}$;

Oscillator dipole model

Susceptibility
$$\chi = \frac{Ne}{m\varepsilon}\left((\upsilon_0 - \upsilon)g(\upsilon) + ig(\upsilon)\right)$$

Lorentzian
$$g(\upsilon) = \frac{\Delta\upsilon_h/2\pi}{(\upsilon-\upsilon_0)^2 + (\Delta\upsilon_h/2)^2}$$

Normalized Lorentzian
$$\tilde{g}(\upsilon) = \frac{g(\upsilon)}{g(\upsilon_0)} = \frac{(\Delta\upsilon_h/2)^2}{(\upsilon-\upsilon_0)^2 + (\Delta\upsilon_h/2)^2}$$

Group velocity $v_g = \frac{d\omega}{dk}$: Cauchy formula for index $n = A + \frac{B}{\lambda^2}$

Prism with apex angle α and minimum deviation angle:
$$n = \sin\left(\frac{\alpha + \delta}{2}\right) / \sin\left(\frac{\alpha}{2}\right)$$

(Dispersive power)
$$\Delta = \frac{D}{\delta} = \frac{n_f - n_c}{n_d - 1}$$
D: dispersion
δ: deviation

Numerical aperture
$NA = \sqrt{n_f{}^2 - n_c{}^2}$

$2NA$ = f# (f-number) = f/D

Imaging:

Thin lens (Lens maker)
$$\frac{1}{f} = \frac{(n_L - n)}{n}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Thin lens
$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

Refraction surface $(n_1 \to n_2)$
$$\frac{n_2 - n_1}{R} = \frac{n_1}{s} + \frac{n_2}{s'}$$

Sign. Conv.
s, s'>0 (Real)
s, s'<0 (Virtual)
R>0 (convex)
R<0 (concave)

(Mirror: $f = -R/2$)

Magnification
$|m| = \left|\frac{h'}{h}\right|;\ m = \frac{-s'}{s}$

Interface ($n_1 \rightarrow n_2$)
$m = -n_1 s'/n_2 s$

Angular mag.
$|m| = \left|\frac{\alpha_M}{\alpha_{unaided}}\right| = \left|\frac{np}{s}\right|$

2-Lens system (L)
$\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2}$

Achromatic Doublet
$f_1 V_1 + f_2 V_1 = 0$

ABCD matrices $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ (det[.]=1)

$\begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} r_1 \\ r_1' \end{pmatrix}$

ABCD law for Gaussian beams
$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}$$

Paraxial ray tracing matrices. (ABCD)

Thin lens (mirror R)
$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$

Interface $n \rightarrow n'$
$\begin{pmatrix} 1 & 0 \\ 0 & \frac{n}{n'} \end{pmatrix}$

Propagation
$\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$

Refractive surface
$n \rightarrow n'$
$\begin{pmatrix} 1 & 0 \\ \frac{n-n'}{n'R} & \frac{n}{n'} \end{pmatrix}$

R>0 (convex)
R<0 (concave)

Optical Resonator $\quad -1 \le cos\theta = (A+D)/2 \le 1$

Cavity round trip
$$r_N = \frac{1}{sin\theta}\{[A\sin(N\theta) - \sin(N-1)\,\theta]r_{in} + B\sin(N\theta)\,r_{in}'\}$$
$$r_N' = \frac{1}{sin\theta}\{C\sin(N\theta)\,r_{in} + [D\sin(N\theta) - \sin(N-1)\,\theta]r_{in}'\}$$

q *parameter in a cavity*
$$\frac{1}{q} = -\frac{A-D}{2B} - i\frac{\sqrt{1-\left(\frac{A+D}{2}\right)^2}}{B}$$

Trigonometry

$\sin^2\theta + \cos^2\theta = 1$

$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

$\cos 2\theta = 1 - 2\sin^2\theta;\quad \sin 2\theta = 2\sin\theta\cos\theta$

$\sin\theta = \cos(\pi/2 - \theta);\quad \cos\theta = \sin(\pi/2 - \theta)$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1}{\cot\theta}$

Useful relations

$\frac{d\sin\theta}{d\theta} = \cos\theta$

$\frac{d\cos\theta}{d\theta} = -\sin\theta$

$d\ln w/_{dx} = w'/w$

$\frac{dw^a}{dx} = aw^{a-1}w'$

Paraxial approx.:
$\tan\theta \approx \sin\theta \approx \theta$

Euler:
$e^{i\theta} = \sin\theta + i\cos\theta$

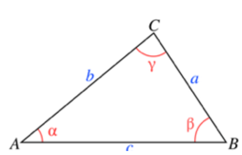
L'Hôpital's rule
If $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) \rightarrow 0, \pm\infty$
Then:
$\lim_{x\to a} f(x)/g(x) = \lim_{x\to a} f'(x)/g'(x)$

Law of cosines

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$a^2 = b^2 + c^2 - 2ab\cos\alpha$

$b^2 = a^2 + c^2 - 2ab\cos\beta$



Physical Constants

$c = 2.998 \times 10^8$ m·s^{-1}	$h = 6.63 \times 10^{-34}$ J·s	$e = 1.6 \times 10^{-19}$ C
$k_B = 1.380 \times 10^{-23}$ J·K^{-1}	$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m	$m_e = 9.1 \times 10^{-31}$ kg

1G = 10^{-4} T, 1 eV = 1.602 x 10^{-19} J, 1 dyne = 10^{-5} N, 1 erg = 10^{-7} J

Near distance (near point) of normal eye: 250 mm