Formula Sheet PHYC/ECE 463 Advanced Optics 1 UNM (MT1)

Harmonic wave: $E = Re\{E_0 \exp(-i\omega t + ik\cdot r + \phi)\}$ $k = \frac{n\omega}{c} = \frac{2\pi n}{\lambda_0}$; $\frac{1}{\mu_0\mu\varepsilon_0} = c^2$

Poynting vector $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ Irradiance $I = \langle S\rangle = \frac{\varepsilon_0 nc}{2}|E_0|^2$

Harmonic waves: $\vec{B} = \frac{1}{\omega}\vec{k}\times\vec{E} = \frac{1}{c}\hat{k}\times\vec{E}$ ($\hat{k}$ unit vector along $\vec{k}$);

Snell: $n_i sin\theta_i$=$n_t sin\theta_t$ critical/pol. angles: $sin\theta_C$=$tan\theta_B = n_2/n_1$

De Broglie Wavelength	Relativistic Momentum	Relativistic Velocity	Relativistic mass
$\lambda = h/p$	$p = \sqrt{E^2/c^2 - m^2c^2}$	$v = \frac{pc^2}{E} = c\sqrt{1 - m^2c^4/E^2}$	$m_{rel} = \frac{m}{\sqrt{1 - v^2/c^2}}$ (Photon: $m = 0$)

Huygens: Each point on the surface of a wavefront may be a source of spherical waves, which themselves progress with the speed of light in the medium ($v = c/n$) and whose envelope at a later time constitutes the new wavefront.

Fermat: Light travels between two points along the path that requires the least time,(or extremizes the optical path length), as compared to other nearby paths.

Imaging:

Thin lens (Lens maker)
$$\frac{1}{f} = \frac{(n_L - n)}{n}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Thin lens
$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$
(Mirror: $f = -R/2$)

Refraction surface ($n_1 \to n_2$)
$$\frac{n_2 - n_1}{R} = \frac{n_1}{s} + \frac{n_2}{s'}$$

Sign. Conv.
s, s'>0 (Real)
s, s'<0 (Virtual)
R>0 (convex)
R<0 (concave)

Magnification
$|m| = \left|\frac{h'}{h}\right|$; $m = \frac{-s'}{s}$

Interface ($n_1 \to n_2$)
$m = -n_1 s'/n_2 s$

Angular mag.
$$|m| = \left|\frac{\alpha_M}{\alpha_{unaided}}\right| = \left|\frac{np}{s}\right|$$

2-Lens system (L)
$$\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2}$$

Achromatic Doublet
$$f_1V_1 + f_2V_1 = 0$$

ABCD matrices $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ (det[.]=1) $\begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} r_1 \\ r_1' \end{pmatrix}$

Paraxial ray tracing matrices. (ABCD)

Thin lens (mirror R)
$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

Interface $n \to n'$
$$\begin{pmatrix} 1 & 0 \\ 0 & \frac{n}{n'} \end{pmatrix}$$

Propagation
$$\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$$

Refractive surface $n \to n'$
$$\begin{pmatrix} 1 & 0 \\ \frac{n - n'}{n'R} & \frac{n}{n'} \end{pmatrix}$$

R>0 (convex)
R<0 (concave)

Optical Resonator

Cavity round trip

$$r_N = \frac{1}{sin\theta}\{[A\sin(N\theta) - \sin(N-1)\,\theta]r_{in} + B\sin(N\theta)\,r'_{in}\}$$

$$-1 \le cos\theta = (A+D)/2 \le 1$$

$$r'_N = \frac{1}{sin\theta}\{C\sin(N\theta)\,r_{in} + [D\sin(N\theta) - \sin(N-1)\,\theta]r'_{in}\}$$

GRIN system

Refractive index

$n(r) \approx n_0[1 - r^2/2l^2\,]$

r.- distance from OA

l.- rate of change of n

ABCD

$$\begin{pmatrix} \cos(d/l) & l\sin(d/l) \\ -1/l\ \sin(d/l) & \cos(d/l) \end{pmatrix}$$

d.- prop distance

Trigonometry

$\sin^2\theta + \cos^2\theta = 1$

$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

$\cos 2\theta = 1 - 2\sin^2\theta$; $\sin 2\theta = 2\sin\theta\cos\theta$

$\sin\theta = \cos(\pi/2-\theta)$; $\cos\theta = \sin(\pi/2-\theta)$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{\cot\theta}$

Useful relations

$\dfrac{d\sin\theta}{d\theta} = \cos\theta$

$\dfrac{d\cos\theta}{d\theta} = -\sin\theta$

$d\ln w/_{dx} = w'/w$

$\dfrac{dw^a}{dx} = aw^{a-1}w'$

Paraxial approx.:

$\tan\theta \approx \sin\theta \approx \theta$

Euler:

$e^{i\theta} = \sin\theta + i\cos\theta$

L'Hôpital's rule

If $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) \to 0, \pm\infty$

Then:

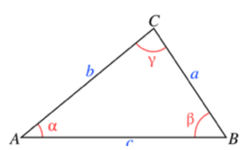
$\lim_{x\to a} f(x)/g(x) = \lim_{x\to a} f'(x)/g'(x)$

Law of cosines

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$a^2 = b^2 + c^2 - 2ab\cos\alpha$

$b^2 = a^2 + c^2 - 2ab\cos\beta$



Physical Constants

$c=2.998 \times 10^8$ m·s^{-1}	$h=6.63 \times 10^{-34}$ J·s	$e=1.6 \times 10^{-19}$ C
$k_B=1.380 \times 10^{-23}$ J·K^{-1}	$\varepsilon_0=8.85\times10^{-12}$ F/m	$m_e=9.1 \times 10^{-31}$ kg

1G = 10^{-4} T, 1 eV = 1.602 x 10^{-19} J, 1 dyne = 10^{-5} N, 1 erg = 10^{-7} J