Formula Sheet PHYC/ECE 463 Advanced Optics 1 UNM (FINAL)

Harmonic wave: $E = Re\{E_0 \exp(-i\omega t + ik \cdot r + \phi)\}$ $k = \frac{n\omega}{c} = \frac{2\pi n}{\lambda_0}$; $\frac{1}{\mu_0 \mu \varepsilon_0} = c^2$

Poynting vector $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ $\vec{B} = \frac{1}{\omega} \vec{k} \times \vec{E} = \frac{1}{c} \hat{k} \times \vec{E}$ ($\hat{k}$ unit vector $\parallel \vec{k}$); Irradiance: $I = \langle S \rangle = \frac{\varepsilon_0 nc}{2} |E_0|^2$

Hermite-Gaussian Beams

$$\frac{E(x,y,z)}{E_0} = H_m\left(\frac{\sqrt{2}x}{w(z)}\right) H_p\left(\frac{\sqrt{2}y}{w(z)}\right) \frac{w_0}{w(z)} exp\left(-i\frac{kr^2}{2q(z)}\right) \times exp\left(-i\left[kz - (1+m+p)tan^{-1}\left(\frac{z}{z_0}\right)\right]\right), \begin{cases} H_0(u) = 1; H_1(u) = 2u \\ H_2(u) = 2(2u^2 - 1) \end{cases}$$

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi n w^2(z)};\quad w^2(z) = w_0^2\left(1 + \frac{z^2}{z_0^2}\right);\quad R(z) = z\left(1 + \frac{z_0^2}{z^2}\right);\quad z_0 = \frac{\pi n w_0^2}{\lambda_0};$$

Divergence $\theta_0 = \frac{2\lambda}{\pi\omega_0}$

Interference

phaseshift $\delta = k\Delta$

Interference Visibility $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$

Young $\Delta = a \sin\theta$

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\delta)$ - Resulting intensity from the interference between two waves with a δ phase-shift.

$\Delta_n = 2n_f t \times \cos\theta_t$ - Optical path length difference in single film

Fabry-Perot

$\delta = 2kd$ $T = \frac{1}{1 + F_C \sin^2(\delta/2)}$ *(Coefficient of Finess)* $F_C = \frac{4r^2}{(1-r^2)^2}$ $Finess = F = \frac{\pi r}{1-r^2} = \frac{\delta_{fsr}}{2\delta_{1/2}} = \frac{d_{fsr}}{2\Delta d_{1/2}}$	$\delta_{1/2} = \frac{\pi}{Finess}$ $\delta_{FSR} = 2\pi$ *Resoling power* $= \frac{\lambda}{\Delta\lambda_{\min}} = \frac{2dF}{\lambda} = mF$ $Q == \omega_0 \tau_p \approx \frac{\nu}{2\Delta\nu_{1/2}} = F \times \frac{\nu}{\Delta\nu_{fsr}}$

Michelson

$\Delta_p = 2d \times \cos\theta$

$|\Delta\theta| = \frac{\lambda \Delta m}{2d \sin\theta}$

$\Delta m = \frac{2\Delta d}{\lambda}$

Farby Perot (time/Energy storage)

$\tau_{RT} = 1/\nu_{FSR} = 2d/c$ $\qquad \tau_p = \Gamma^{-1} = \tau_{RT}/2(1-r)$ $\qquad E = E_0 \exp(-\Gamma t/2)$

Fraunhoffer Diffraction

$$E_p = \int_{Aperture} dE_p = \frac{E_L}{r} e^{i(kr - \omega t)} = \int e^{ik\, s \sin\theta} ds$$

-- ***Fraunhofer diffraction*** --

$L >> \frac{\text{area of aperture}}{\lambda}$ *Far-field condition*

From N equally spaced (period = a) slits (width= b)

$I = I_0 \sin c^2(\beta)\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$ $\qquad \beta = 1/2(kb \sin\theta)$ $\quad \alpha = 1/2(ka \sin\theta)$

$\alpha = \frac{p\pi}{N}$ Principal maxima: $p = 0, \pm N, \pm 2N, \pm 3N$. secondary minima p= all other integer values

$m\lambda = a \sin\theta_m$ Angular location of principal maxima relative to the axis of symmetry

Diffraction grating

---------------------------- Diffraction grating

$m\lambda = a(\sin\theta_i + \sin\theta_m)$ General grating equation

$\lambda_{FSR} = \dfrac{\lambda_1}{m}$ $\quad R = \dfrac{\lambda}{(\Delta\lambda)_{\min}} = mN = \dfrac{W\sin\theta_m}{\lambda}$ $\quad D = \dfrac{m}{a\cos\theta_m}$ $\quad linear - dispersion = fD$

$m\lambda = a[\sin\theta_i + \sin(2\theta_b - \theta_i)]$ Blazed grating equation (θ_b is the blaze angle and a is the grating period)
(*Littrow*: $\theta_b = \theta_i$, Normal incidence: $\theta_i = 0$)

Polarization

Linear Polarizer

$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ (TA horizontal) $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ (TA vertical) $\dfrac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (TA 45°)

Pol. Rotator $(\theta \to \theta + \beta)$

$\begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix}$

Retarder (δ relative phaseshift)

$\begin{pmatrix} e^{i\varepsilon_x} & 0 \\ 0 & e^{i\varepsilon_y} \end{pmatrix} = e^{i\varepsilon_x}\begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix}$

QWP (SAV)

$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{pmatrix}$

HWP (SAV)

$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{pmatrix}$

Transform. rotated element ***T***

$\boldsymbol{T}(\phi) = R(-\phi)\,\boldsymbol{T}\,R(\phi)$

$R(\phi) = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}$

Pol. states $|H\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $|V\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $|D\rangle = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$; $|A\rangle = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$; $|LCP\rangle = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$; $|RCP\rangle = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$

Field

$E = \begin{pmatrix} E_{0x} \\ E_{0y}e^{i\varepsilon} \end{pmatrix}$

Pol angle

$\tan 2\alpha = \dfrac{2E_{0x}E_{0y}\cos\varepsilon}{E_{0x}{}^2 - E_{0y}{}^2}$

Birefringence

$\sin 2\chi = \dfrac{2E_{0x}E_{0y}\cos\epsilon}{E_{0x}^2 + E_{0y}^2}$

Linear polarized field

$E_{lin} = E_0\begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$

pol. angle: $tan\theta_B = n_2/n_1$

Trigonometry

$\sin^2\theta + \cos^2\theta = 1$

$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

$\cos 2\theta = 1 - 2\sin^2\theta;\quad \sin 2\theta = 2\sin\theta\cos\theta$

$\sin\theta = \cos(\pi/2 - \theta);\quad \cos\theta = \sin(\pi/2 - \theta)$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{\cot\theta}$

Useful relations

$\dfrac{d\sin\theta}{d\theta} = \cos\theta$

$\dfrac{d\cos\theta}{d\theta} = -\sin\theta$

$d\ln w/_{dx} = w'/w$

$\dfrac{dw^a}{dx} = aw^{a-1}w'$

Paraxial approx.:

$\tan\theta \approx \sin\theta \approx \theta$

Euler:

$e^{i\theta} = \sin\theta + i\cos\theta$

L'Hôpital's rule

If $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) \to 0, \pm\infty$

Then:

$\lim_{x\to a} f(x)/g(x) = \lim_{x\to a} f'(x)/g'(x)$

Physical Constants

$c = 2.998 \times 10^8$ m·s^{-1}	$h = 6.63 \times 10^{-34}$ J·s	$e = 1.6 \times 10^{-19}$ C
$k_B = 1.380 \times 10^{-23}$ J·K^{-1}	$\varepsilon_0 = 8.85\times10^{-12}$ F/m	$m_e = 9.1 \times 10^{-31}$ kg

1G = 10^{-4} T, 1 eV = 1.602 x 10^{-19} J, 1 dyne = 10^{-5} N, 1 erg = 10^{-7} J