

# Homework 3

**(Due Date: Monday, Feb. 16)**

Note: when submitting your solutions include procedures and derivations.

**Problem 1.**- Given the function  $f(u, v)$  which depends on variables  $u$  and  $v$  with variances  $\sigma_u^2$  and  $\sigma_v^2$ , respectively, obtain the variance  $\sigma_f^2$  of  $f(u, v)$  in terms of the variances  $\sigma_u^2$  and  $\sigma_v^2$  and the covariance  $\sigma_{uv}$  of  $u$  and  $v$  for the following cases where  $a$  and  $b$  are positive constants:

(a)  $f = au \pm bv$

(b)  $f = \pm auv$

(c)  $f = \pm au/v$

(d)  $f = au^{\pm b}$

(e)  $f = ae^{\pm bu}$

(f)  $f = a \ln(\pm bu)$

Find the uncertainty  $\sigma_x$ , in “ $x$ ” as a function of the uncertainties  $\sigma_u$ , and  $\sigma_v$ , in  $u$  and  $v$  for the following functions:

(g)  $x = \frac{1}{2(u+v)}$

(h)  $x = \frac{1}{2(u-v)}$

(i)  $x = \frac{1}{u^2}$

(j)  $x = uv^2$

(k)  $x = u^2 + v^2$

Note: for cases from (a) to (c) assume that the variables  $u$  and  $v$  are correlated. For cases from (g) to (k) assume that the variables  $u$  and  $v$  are not correlated.

**Problem 2:**

The Doppler shift describes the frequency change when a source of sound waves of frequency “ $f$ ” moves with a velocity “ $v$ ” towards an observer at rest as  $\Delta f = fv/(u - v)$ , where “ $u$ ” is the velocity of sound. Determine the Doppler shift and its uncertainty for the situation when

$u = (332 \pm 8)$  m/s

$v = (0.123 \pm 0.003)$  m/s

$f = (1000 \pm 1)$  m/s

What is the quantity that contributes the least and the most to the uncertainty of the Doppler shift?

**Problem 3.**- The period  $T$  of a pendulum is related to its length by the relation

$$T = 2\pi \sqrt{\frac{L}{g}},$$

where  $g$  is the gravitational acceleration. Suppose you are measuring  $g$  from the period and length of the pendulum. You have measured the length of the pendulum to be  $1.1325 \pm 0.0014$  m. You independently measure the period to within an uncertainty of 0.06%, that is  $\sigma_T/T = 6 \times 10^{-4}$ . What is the

fractional uncertainty in  $g$  ( $\sigma_g/g$ ), assuming that the uncertainties in  $L$  and  $T$  are independent and random?

**Problem 4.**- Niels Bohr showed that the energy ( $E_n$ ) of the quantum states of a Hydrogen atom are given by:

$$E_n = -2\pi^2 \frac{m e^4}{h^2} \frac{1}{n^2}$$

where  $m$  is the mass of the electron,  $e$  is charge and  $h$  is Planck's constant.  $n$  is the Principle Quantum Number and is an exact number. Suppose the relative error ( $\sigma_u/u$ ) in each of the measured quantities  $m$ ,  $e$  and  $h$  is:

<u>Quantity</u>	<u>Rel. Error</u>
$m$	0.001
$e$	0.002
$h$	0.0001

What is the relative error in energy of the third quantum state? Assume all the errors are statistically determined standard deviations of the mean.